Section 5.2

The Algebra of Functions

Model 1 Operations of Functions

Given Functions	x = −5	x = 0
$f(x) = x + 1$ $g(x) = x^2 - 5$ $h(x) = x^2 - 1$	$f(-5) = -5 + 1 = -4$ $g(-5) = (-5)^2 - 5 = 20$ $h(-5) = (-5)^2 - 1 = 24$	$f(0) = 1$ $g(0) = -5$ $h(0) = -1$

Addition	
$(f + g)(-5) = -4 + 20 = 16$ $(g + h)(-5) = 20 + 24 = 44$	$(f + g)(x) = (x + 1) + (x^2 - 5) = x^2 + x - 4$ $(g + h)(x) = (x^2 - 5) + (x^2 - 1) = 2x^2 - 6$
Subtraction	
$(f - g)(0) = 1 - (-5) = 6$ $(h - f)(0) = -1 - (1) = -2$	$(f - g)(x) = (x + 1) - (x^2 - 5) = -x^2 + x + 6$ $(h - f)(x) = (x^2 - 1) - (x + 1) = x^2 - x - 2$
Multiplication	
$(f \cdot g)(-5) = -4 \cdot 20 = -80$ $(h \cdot g)(0) = -1 \cdot -5 = 5$	$(f \cdot g)(x) = (x + 1)(x^2 - 5) = x^3 + x^2 - 5x - 5$ $(h \cdot f)(x) = (x^2 - 1)(x + 1) = x^3 + x^2 - x - 1$
Division	
$\left(\frac{f}{g}\right)(-5) = \frac{-4}{20} = \frac{-1}{5}$ $\left(\frac{g}{f}\right)(-5) = \frac{20}{-4} = -5$	$\left(\frac{f}{g}\right)(x) = \frac{x + 1}{x^2 - 5}$ $\left(\frac{h}{f}\right)(x) = \frac{x^2 - 1}{x + 1} = \frac{(x + 1)(x - 1)}{x + 1} = x - 1$

1. Use the information in Model 1 to answer the following:

   a) What is $f(x)$? ___________

   b) What is the value of $f(0)$? ___________

   c) What is $g(x)$? ___________

d) What is the value of $g(0)$? __________

e) What is the value of $(f - g)(0)$? __________

f) Write an expression for $(f - g)(0)$ in terms of $f(0)$ and $g(0)$.

g) In view of your answer above, find the value of $(g - f)(0)$.

2. Locate the expression $(g + h)(-5)$ in Model 1.

a) What is the value of $(g + h)(-5)$? __________

b) Explain where the 20 and 24 come from in the addition.

c) What is the simplified expression of $(g + h)(x)$ in Model 1?

d) Use the simplified expression above to confirm your answer to part a.

3. Locate the expression $\left(\frac{f}{g}\right)(-5)$ in Model 1.

a) What is the value of $\left(\frac{f}{g}\right)(-5)$?

b) What is the relationship between the values $\left(\frac{f}{g}\right)(-5)$ and $\left(\frac{g}{f}\right)(-5)$?

c) Find the expression for $\left(\frac{g}{f}\right)(x)$ using the functions in Model 1.

4. Describe two different ways to find the value of $(h \cdot g)(0)$.

5. Find the following values using the functions from Model 1.

a) $(f + g)(0)$

b) $(f \cdot h)(-5)$

c) $(f - h)(0)$

d) $\left(\frac{g}{h}\right)(0)$

e) $(f - g)(2)$

6. Find the simplified expressions for the following using the functions from Model 1.

a) $(f + h)(x)$

b) $(g - h)(x)$

c) $(f \cdot h)(x)$

d) $\left(\frac{f}{h}\right)(x)$

7. In summary of the operations presented in Model 1, write a general expression in terms of $f(x)$ and $g(x)$ for each of the following:

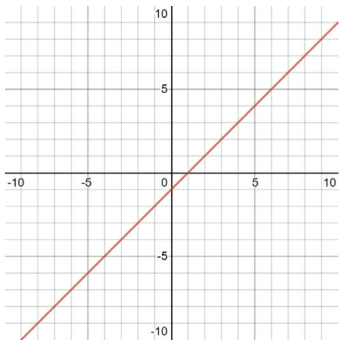
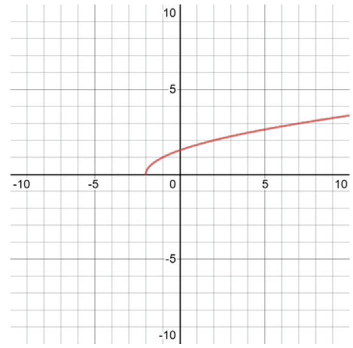
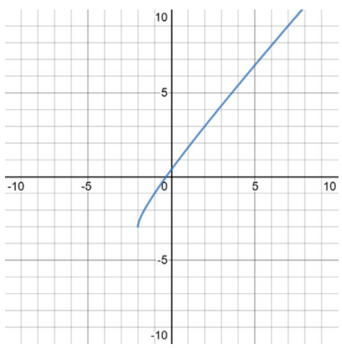
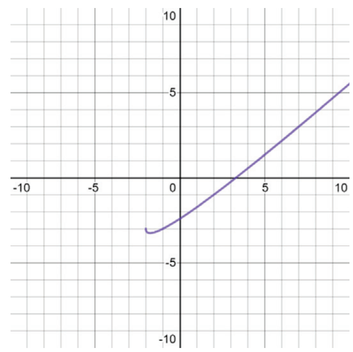
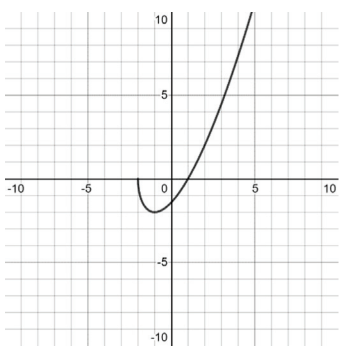
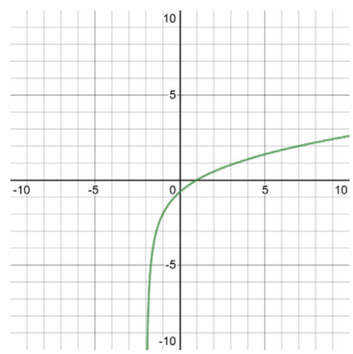
$(f + g)(x) =$

$(f - g)(x) =$

$(f \cdot g)(x) =$

$\left(\frac{f}{g}\right)(x) =$

Model 2 Finding Domains with Function Operations

Function & Domain	Graph	Function & Domain	Graph
$f(x) = x - 1$ D: $(-\infty, \infty)$		$g(x) = \sqrt{x+2}$ D: $[-2, \infty)$	
$(f+g)(x)$ $= x - 1 + \sqrt{x+2}$ D: $[-2, \infty)$		$(f-g)(x)$ $= x - 1 - \sqrt{x+2}$ D: $[-2, \infty)$	
$(fg)(x)$ $= (x-1)\sqrt{x+2}$ D: $[-2, \infty)$		$\left(\frac{f}{g}\right)(x)$ $= \dfrac{x-1}{\sqrt{x+2}}$ D: $(-2, \infty)$	

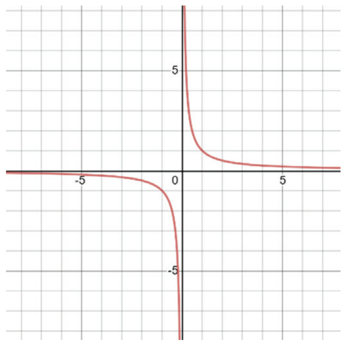
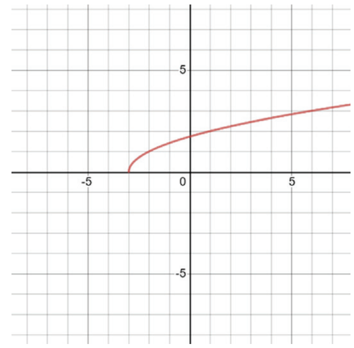
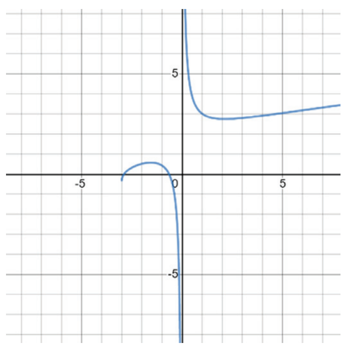
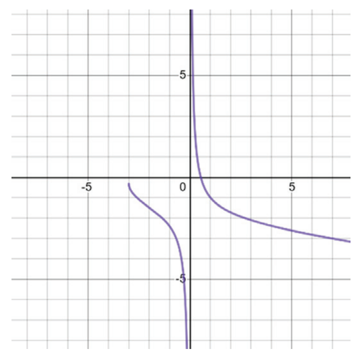
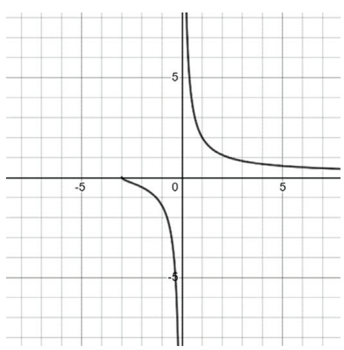
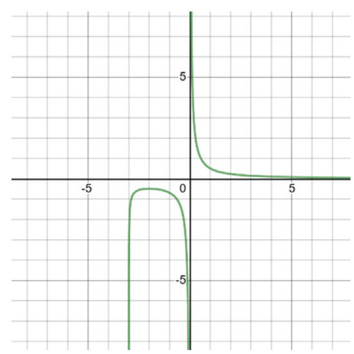
8. Which original function, $f(x)$ or $g(x)$, has a restricted domain? Explain the reason for the restriction.

9. Is the domain for the function operations similar to the domain of $f(x)$ or the domain of $g(x)$?

10. Which of the four operations has a different domain than the others? Describe the difference and explain why.

11. Discuss as a team, and explain how can a graph help to verify the domain.

12. Now consider the following six functions and graphs.

Function	Graph	Function	Graph
$f(x) = \frac{1}{x}$ D: $(-\infty, 0) \cup (0, \infty)$		$g(x) = \sqrt{x+3}$ D: $[-3, \infty)$	
$(f+g)(x)$ D: __________		$(f-g)(x)$ D: __________	
$(fg)(x)$ D: __________		$\left(\frac{f}{g}\right)(x)$ D: __________	

a) What value is excluded in the domain of $f(x)$ and why?

b) What values are excluded from the domain of $g(x)$ and why?

13. Based on the graph in Question 12, what values appear to be excluded from the domain of $(f+g)(x)$?

14. Find $(f+g)(x)$ using the functions from Question 12 and write the domain.

15. Which two other function operations have the same domain as $(f+g)(x)$? Present your team's justification.

16. The domains for $(f+g)(x), (f-g)(x)$, and $(fg)(x)$ will always be the same. These domains come from the *intersection* of the domain of $f(x)$ and $g(x)$. Fill in the missing domains for $(f+g)(x), (f-g)(x)$, and $(fg)(x)$ in Question 12.

17. The domain for $\left(\frac{f}{g}\right)(x)$ also comes from the intersection; however, we must consider additional restrictions for what makes the denominator function zero.

a) In the expression $\left(\frac{f}{g}\right)(x)$, what is the denominator function? __________

b) Find $\left(\frac{f}{g}\right)(x)$ using the functions from Question 12 and write the domain.

c) What additional value is excluded from the domain for $\left(\frac{f}{g}\right)(x)$?

d) Fill in the missing domain in the table for $\left(\frac{f}{g}\right)(x)$ in Question 12.

18. Given $f(x) = \dfrac{-3}{x-1}$ and $g(x) = \dfrac{3}{5-x}$, find:

a) $(f+g)(x)$ and its domain

b) $(f-g)(x)$ and its domain

c) $(f \cdot g)(x)$ and its domain

d) $\left(\frac{f}{g}\right)(x)$ and its domain

Exercises

Let $f(x) = x^2 - 9$, $g(x) = x - 3$, and $h(x) = 2x$. Find each of the following:

1. $(f + g)(2)$
2. $(f - g)(-3)$
3. $(f - h)(0)$
4. $(fh)(1)$
5. $\left(\frac{g}{h}\right)(6)$
6. $(h + g)(-\frac{1}{4})$
7. $(g + f)(x)$
8. $(f - h)(x)$
9. $(fg)(x)$
10. $\left(\frac{f}{g}\right)(x)$
11. $(g + h)(x)$
12. $(gh)(x)$

Let $f(x) = \dfrac{3}{x-4}$ and $g(x) = \dfrac{1}{x^2 - 16}$. Find each of the following:

13. $(f + g)(x)$ and its domain.
14. $(f - g)(x)$ and its domain.
15. $(fg)(x)$ and its domain.
16. $\left(\frac{f}{g}\right)(x)$ and its domain.

Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x-1}$. Find each of the following. Be sure to rationalize all denominators.

17. $(f + g)(x)$ and its domain.
18. $(f - g)(x)$ and its domain.
19. $(fg)(x)$ and its domain.
20. $\left(\frac{f}{g}\right)(x)$ and its domain.
21. $\left(\frac{g}{f}\right)(x)$ and its domain.