Functions 1: Introduction to Functions

Model 1: Zoo and Aquarium Prices

The tables below represent admission prices (output) given a person's age (input).

Zoo Admission Prices	
Age	**Price of admission**
5 & under	$3
6 -12	$6
13 -64	$12
65 years & over	$6

Aquarium Admission Prices	
Age	**Price of admission**
Infants (2 & under)	free
Kids 2-17	$5
Adults (18 & over)	$10

Construct Your Understanding Questions (to do in class)

1. How much does it cost for a 4 year old to enter the…
   a. Zoo? b. Aquarium?

2. How much does it cost for a 2 year old to enter the…
   a. Zoo? b. Aquarium?

3. A mother is standing in line to buy tickets for her and her two year old to enter the Aquarium. She looks at the prices on the board and gets her $10 bill ready to pay for both of them. At the window, the person at the counter requests $15 for both tickets.
   a. Referencing the table, explain how each person decided on the price to pay for the tickets.

   b. Would it make sense for an aquarium to have this kind of pricing scheme? If not, how could this be avoided?

4. Do Zoo admission prices have any similar problems? Explain why or why not.

5. According to the definition in Summary Box F1.1:
   a. Do the Zoo admission prices represent a **function**?
   Explain your reasoning.

   b. Do the Aquarium admission prices represent a **function**?
   Explain your reasoning.

Summary Box F1.1: Definition of a Function

A **function** is a rule such that for each input there is exactly one output that corresponds to it. We then say the output is a **function of** the input.

6. (Check your work) Suggest a change to the Aquarium admission prices that would make admission price a function of age.

7. For each of the following pairs, identify the input and output that would create a function. Fill in the blanks and use the definition in Summary Box F1.1 to provide a justification for your choice.

   a. In each of the blanks below choose one: **Student ID number** or **GPA**.

   For each _____, there is exactly one _____ that corresponds to it.

   Therefore, _____(output) is a function of _____(input).

   b. In each of the blanks below choose one: **Altitude** or **GPS coordinates of a place on earth**.

   For each _____, there is exactly one _____ that corresponds to it.

   Therefore, _____ (output) is a function of _____(input).

c. In each of the blanks below choose one: **Amount in US Dollars** or **Amount in Euros**

For each _____, there is exactly one _____ that corresponds to it.

Therefore, _____(output) is a function of _____(input).

8. (Check your work) For one of the pairs above there is more than one possible choice for the input. Identify which one and explain your reasoning.

9. Which of the following tables represent functions? Cross out the ones that are not functions.

Input	Output
6	1
8	5
6	7
4	9

Input	Output
1	6
5	8
7	6
9	4

Input	Output
2	1
3	1
4	1
5	1

Input	Output
1	4
1	8
1	12
1	16

10. (Check your work) We can describe a set by listing its elements between curly brackets, "{" and "}", listing each element only once.
    a. Verify that {4,6,8} is the set of outputs for one of the tables above representing a function. Place a star above that table. Note that even if the elements appear multiple times we list it only once.
    b. Use set notation to express the set of inputs of this function.

       Set of inputs =

11. (Check your work) Functions can have repeated outputs. In fact, it is possible for a function to have the same output for each input. Those functions are called constant functions, and their set of outputs has only one element.
    a. Among the two tables you identified as functions above, circle the constant function.
    b. Use set notation to express the set of inputs and the set of outputs for the constant function.

       Set of inputs =

       Set of outputs =

Model 2: Functions, domain and range

Each function below (labeled f, g, h and q) takes an input, and then produces a corresponding output, forming a unique pair in which the input and the output are related to one another by some rule that the function uses to operate.

Functions	Examples	
Function f Domain: Set of inputs = English words that written backwards yield an English word Range: Set of outputs = English words that written backwards yield an English word	BONK $\rightarrow$ [f] $\rightarrow$ KNOB Input Output $f(\text{BONK}) = \text{KNOB}$	PIN $\rightarrow$ [f] $\rightarrow$ [ ] Input Output $f($ ____ $) =$ _____
Function g $x \rightarrow$ [g] $\rightarrow \sqrt{x}$ Input Output Domain: Set of all real number inputs that produce a real number as an output Range: Set of all real outputs produced by the valid real number inputs	$4 \rightarrow$ [g] $\rightarrow 2$ Input Output $g(4) = 2$	$25 \rightarrow$ [g] $\rightarrow$ [ ] Input Output $g($ ____ $) =$ _____
Function h $x \rightarrow$ [h] $\rightarrow x^2$ Input Output Domain: Set of all real number inputs that produce a real number as an output Range: Set of all real outputs produced by the valid real number inputs	$-3 \rightarrow$ [h] $\rightarrow 9$ Input Output $h($ ____ $) = 9$	$6 \rightarrow$ [h] $\rightarrow$ [ ] Input Output $h($ ____ $) =$ _____
Function q $x \rightarrow$ [q] $\rightarrow \frac{1}{\sqrt{x}}$ Input Output Domain: Set of all real number inputs that produce a real number as an output Range: Set of all real outputs produced by the valid real number inputs	$\frac{1}{9} \rightarrow$ [q] $\rightarrow 3$ Input Output $q\left(\frac{1}{9}\right) =$ _____	$4 \rightarrow$ [q] $\rightarrow$ [ ] Input Output $q($ _____ $) =$ _____

Construct Your Understanding Questions (to do in class)

12. The notation $f(\text{BONK})$ represents the output of function f with an input of BONK. For this particular function, $f(\text{BONK}) = \text{KNOB}$.
    a. Translate the following sentence: Steve put the $f(\text{TOP})$ on the table.

    b. Using the functions in Model 2, fill in the 4 *boxes* labeled "output" in the last column of the table and all the other blanks in the table.

13. Suppose we have a function k which takes as input **Student ID number** and gives as output the **GPA** of that student.

    a. Is $k(0523621)$ an **input** or an **output** of the function? Circle one and explain your choice.

    b. Is $k(0523621)$ a **student ID number** or a **GPA**? Circle one and explain your choice.

    c. Which of the following is the correct interpretation of $k(0523621)$?

        i. $k(0523621)$ is the student ID with ID number 0523621.
        ii. $k(0523621)$ is the GPA of the student with ID number 0523621.

14. Circle the words below that are in the domain of function f in Model 2 and explain why the remaining words are not in the domain. (Be sure to use the description of the function that appears in the table.)

    RAW APPLE TULIP SMART STAR TREE WINDOW WARTS

15. Based on the functions and the descriptions of their domain in Model 2, which one of the following statements is false? Explain your answer.

    a. $\sqrt{-2}$ is not a real number, so -2 is not in the domain of g.
    b. $q(3) = \frac{1}{\sqrt{3}}$
    c. $g(0) = 0$
    d. $f(123) = 321$

16. (Check your work) Recall that $\sqrt{0} = 0$. If necessary, revise your answer to the previous question.

17. If a given input is not in the domain of the function, we say that the function is **undefined** for that input. Fill in the outputs for functions g, h and q from Model 2 using the word **undefined** where necessary.

a. $g(100) =$ _____ d. $h(100) =$ _____ g. $q(100) =$ _____

b. $g(0) =$ _____ e. $h(0) =$ _____ h. $q(0) =$ _____

c. $g(-100) =$ _____ f. $h(-100) =$ _____ i. $q(-100) =$ _____

18. (Check your work) Answer the following questions.

a. 0 is not in the domain of exactly one of the functions g, h or q. Which one?

b. -100 is not in the domain of exactly two of the functions g, h or q. Which two?

19. Let x be a real number. As illustrated, draw arrows in order to write the domain and range of each function two different ways. One example is done for you. Be careful: Some of them may not be used at all and some may be used more than once!

Recall that the square brackets "[" or "]" are used in interval notation to indicate that an endpoint ***is*** included in the interval, and parentheses "(" or ")" are used if the endpoint ***is not*** included in the interval. We always use a parenthesis next to ∞ or $-\infty$.

Domain of function g	$\{x : x > 0\}$	$(-\infty, \infty)$
Domain of function h	All real numbers	$[0, \infty)$
Domain of function q	$\{x : x \geq 0\}$	$(0, \infty)$

Range of function g	$\{y : y > 0\}$	$(-\infty, \infty)$
Range of function h	All real numbers	$[0, \infty)$
Range of function q	$\{y : y \geq 0\}$	$(0, \infty)$

Check your answers with at least one other group.

20. Fill out the reasoning needed to find the domain of the functions below. The first example is done for you.

Function	Reasoning	Conclusion
$H(x)=\frac{1}{x-4}$	H is defined whenever $x-4\neq 0$. That is, whenever $x\neq 4$.	Domain of H : $(-\infty,4)\cup(4,\infty)$
$G(x)=\frac{6}{x^2-4}$		Domain of G : $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$
$F(x)=\frac{1}{x^2+4}$		Domain of F : $(-\infty,\infty)$

21. Consider the following functions.
    - Identify the three functions for which the domain is all real numbers.
    - For each of the other three, use interval notation to describe the domain.

a. $f(x) = 2x + 1$

b. $g(t) = \sqrt{t+3}$

c. $h(s) = \sqrt{s^2 + 1}$

d. $p(u) = u^3 - 125$

e. $q(v) = \dfrac{1}{\sqrt{v-1}}$

f. $f(x)$ is the area of a square with side length x (in cm)

22. Let a be a real number. For the function $g(x) = x^2 - x$ find and simplify…

    a. $g(3) =$

    b. $g(2a) =$

    c. $g(a+2) =$

    d. $g(a) + 2 =$

23. (Check your work) Two of the answers in the previous question are $4a^2 - 2a$ and $a^2 + 3a + 2$. Circle them.

Notes